

Seminar 5: Russell “On Denoting”, Part 1

Russell’s on “On Denoting,” written and published in 1905 become one of the most celebrated and influential philosophical articles in the 20th century. He begins with these words.

“By a “denoting phrase” I mean a phrase such as any one of the following: [here the reader should supply quotes around each phrase] a man, some man, any man, every man, all men, the present King of England, the present King of France, the centre of mass of the Solar System at the first instant of the twentieth century, the revolution of the earth round the sun, the revolution of the sun round the earth. Thus a phrase is denoting solely in virtue of its *form*. We may distinguish three cases: (1) A phrase may be denoting, and yet not denote anything; e.g., “the present King of France”. (2) A phrase may denote one definite object; e.g., “the present King of England”. (3) A phrase may denote ambiguously; e.g., ‘a man’ denotes not many men, but an ambiguous man.” (p. 479)

The examples of denoting phrases given illustrate a large class of expressions. In addition to those Russell mentions, it includes: ‘at least one man’, ‘a least two men,’ ‘several men’. ‘many men’, ‘more than 20 but fewer than 50 men’, ‘most men’, and ‘no men’. Today, members of this class are called *generalized quantifiers*, which are given a unified semantic analysis. Russell wasn’t interested in all members of the class. But it will be useful, as we proceed, to attend to the question of what might have to be added to, or subtracted from, his analyses in order to produce a systematic treatment of the whole class. Even at this early stage, a crucial point can be discerned. In including ‘the man’ along with ‘every man’, ‘any man’, ‘all men’, and ‘some man’ Russell is signaling that it functions quite differently from (logically) proper names, and so is not a singular term. This point – which still remains controversial today – was among his most important insights.

In the next paragraph Russell explains why the theory of denoting is important.

“The subject of denoting is of very great importance not only in logic and mathematics, but also in the theory of knowledge. For example, we know that the centre of mass of the Solar System at a definite instant is some definite point, and we can affirm a number of propositions about it, but we have no immediate *acquaintance* with this point, which is only known to us by description. The distinction between *acquaintance* and *knowledge about* is a distinction between the things we have presentations of, and the things we reach by means of denoting phrases. It often happens that we know that a certain phrase denotes unambiguously, although we have no acquaintance with what it denotes; this occurs in the above case of the centre of mass. In perception we have acquaintance with the objects of perception, and in thought we have acquaintance with objects of a more abstract logical character [e.g. universals and propositional functions]; but we do not necessarily have acquaintance with the objects denoted by phrases composed of words with whose meanings we are acquainted. To take a very important instance: There seems no reason to believe that we are ever acquainted with other people’s minds, seeing that these are not directly perceived; hence what we know about them is obtained through denoting. All thinking has to start from acquaintance: but it succeeds in thinking *about* many things with which we have no acquaintance.”(479-80)

The main reason Russell needed a theory of denoting phrases is to understand the role of denoting in thought. He took it to be axiomatic that all propositions we can entertain are composed entirely of things with which we are acquainted. But he also realized that we can think about many things with which we are not, and cannot be, acquainted. This, he believes, is done by means of denoting.

Russell gives priority to thought. The main reason language is important to him is its role in expressing thought. Since he took thought to be private, and restricted by what one can “directly perceive” plus the abstract universals and propositional functions one cognitively apprehends, the primacy of thought carried with it an implicit *linguistic individualism*. For Russell, language is *not* a social institution participation in which expands one’s cognitive reach by enabling one to entertain propositions beyond one’s solitary grasp. It is the expression of capacities the scope of which is fixed by what one is non-linguistically acquainted with. This limited the analyses he took seriously.

This passage also illustrates his famous distinction between *knowledge by acquaintance* and *knowledge by description*. When we are acquainted with something, we can think about it directly. Having it before our minds, we predicate a property of it, thereby entertaining the singular proposition containing both the object and the property. When we are not acquainted with something, the best we can do is to entertain a proposition, which though it doesn't contain the object as a constituent, is nevertheless, somehow, *about* the object. The promise in the passage is that the theory of denoting that Russell is about to propose will explain what denotation is, and how it does this job.

Finally the penultimate sentence of the paragraph hints at how narrow Russell's conception of acquaintance was becoming, and how far his conception of denoting and *knowledge by description* would be forced to expand in order to take up the slack. If, as he seems to suggest, one is never acquainted with another person, then the main burden of connecting us cognitively to the world will have to be born by denoting.

Initial Analyses of Denoting Phrases

According to Russell, [Everything is F] expresses the proposition that predicates the property *being always true* of the propositional function for which the formula [x is F] stands. Russell uses the word 'proposition' loosely in "On Denoting," sometimes to talk about sentences, and sometimes to talk about their meanings--propositions proper. A similar looseness accompanies his use of 'propositional function' for a formula, or for its meaning. Since, at this stage, Russell believed in non-linguistic propositions, I will reconstruct his remarks in those terms. At this stage, he was still a realist about *propositional functions* too – which I will characterize as functions taking entities as arguments and assigning propositions containing those entities as values. So [Everything is F] expresses the proposition that p_F is always true. When $F = \text{'human'}$, (i) p_F is the function that assigns to any object o the proposition *that o is human*, and (ii) p_F is *always true* iff it assigns a *true* proposition to every object o as argument (i.e., for every object o , the proposition *that o is human* is true). [Nothing is F] and [Something is F] mean, respectively, that p_F is never true, and p_F is sometimes true.

In giving this analysis, Russell takes *the property of being always true* to be primitive. From his perspective, it is the quantifier 'every' that is defined in terms of the antecedently understood notion of a propositional function *being always true*. In addition to taking that notion to be primitive, Russell uses it plus negation to define other quantifiers. This is logically elegant, but it leads to implausible psychological complexity when different quantifiers are interspersed at the beginning of an English sentence. Fortunately, this feature isn't essential to his theory; he could, if he wished, take several different quantifiers to express different *primitive* properties of propositional functions.

This is connected to a further point.

"Everything, nothing, and something, are not assumed to have any meaning in isolation, but a meaning is assigned to every proposition in which they occur. This is the principle of the theory of denoting I wish to advocate: that denoting phrases never have any meaning in themselves, but that every proposition in whose verbal expression they occur has a meaning."

The idea that many denoting phrases don't have "meaning in isolation," even though they occur as meaningful grammatical constituents of meaningful sentences is central to Russell's theory. For an expression to fail to have meaning in isolation is for its semantic role to be something other than that of contributing a constituent (its meaning) to the structured complex of constituent that is the proposition expressed by a sentence containing it. When E has *no meaning in isolation*, and S contains E , the structured proposition expressed by S doesn't contain any single constituent that is the meaning of E . Given Russell's definitions of '*nothing*' and '*something*' in terms of '*everything*', one can see how they fit this profile. But what of '*everything*'? Doesn't 'Everything is human' mean, and express the (false) proposition, *that p_F is always true* – with '*everything*' contributing the property

being always true that is predicated of p_F ? If so, then ‘everything’ is a counterexample to Russell’s general rule about denoting phrases not having meaning in isolation.

It is, however, a lonely one, as we can see from Russell’s analysis of the denoting phrases [every F/all Fs/some F/an F]. Although [every F] and [all Fs] are grammatical constituents of the sentences [Every F is G] and [All Fs are G], their contributions to the proposition p these sentences express is not found in any single constituent of p , but is distributed among p ’s constituents. On Russell’s analysis, p predicates *being always true* (contributed by ‘every’/ ‘all’), of the propositional function $p_{if,then}$ expressed by [If x is F, then x is G] – which is not a grammatical constituent of the original sentences at all. What F contributes to $p_{if,then}$ is the property predicated of an object o supplied as argument in the antecedent of the conditional proposition that $p_{if,then}$ assigns to o as value. The case of [no F] is similar, while [some F] and [an F] are treated as variants of the same thing. The proposition expressed by [Some F is G] and [An F is G] is *the negation of the proposition in which being always true is predicated of the propositional function $p_{\sim \&}$ expressed by $[\sim (x \text{ is } F \ \& \ x \text{ is } G)]$, with F contributing to $p_{\sim \&}$ the property predicated of an argument o in the first conjunct of the negated conjunctive proposition that $p_{\sim \&}$ assigns to o as value. *Even before we get to the central case of [the F], the audacity of the analysis is already present in the gulf Russell posits between the grammatical forms of the sentences under analysis and what he takes to be their logical forms, which correspond to the forms of the propositions expressed by those sentences.**

His remarks about the denoting phrase ‘a man’, which he calls *an indefinite description*, warrant further attention. He says, that if it is true that I met a man, then I met some definite man, “but that is not what I affirm”? Presumably, he means that if it is true that he met a man, then there is some man – call him “Sam” – whom he met; but when he asserts or believes that he met a man, what he asserts or believes is not that he met Sam. Rather, he asserts or believes the negation of the claim that a certain conjunctive propositional function is always false; in effect, he asserts or believes that the propositional function is “sometimes true.” This sounds a little abstract, but one gets the picture.

What about the analysis of ‘I met a man’ as, ‘For some x , I met x and x is *human*’? Why isn’t it, ‘For some x , I met x and x is *a man*’? The answer, one imagines, is that if the latter were offered, it would be objected that the analysis had not succeeded in eliminating the phrase – ‘a man’ – under analysis. In offering his own analysis, Russell avoids this potential objection by relying on the adjective ‘human’, which he takes to be synonymous with ‘a man’ when following the copula. But this strategy doesn’t generalize to all indefinite descriptions. For example, the most natural ways of expressing the Russellian the analyses of ‘I saw a tiger’ and ‘I saw a large man’ are: ‘For some x (I saw x & x is a tiger)’ and ‘For some x (I saw x & x is a large man).’ Is it a problem that the indefinite descriptions ‘a tiger’ and ‘a large man’ haven’t be eliminated?

It might not be if an indefinite description [a G] is a predicate rather than a quantifier when it occurs after the copula, as suggested by the examples in (1).

- 1a. John is (isn’t) *a philosopher*
- b. *John is (isn’t) some philosopher.
- c. *John is (isn’t) at least one philosopher.

But it can’t be denied that some sentences containing indefinite descriptions do have quantificational readings, as indicated by the parallel between the sentences in (2) and those in (3) and (4).

- 2a. *A large man* will meet you.
 - b. You will meet *a large man* on the bridge.
- 3a. Some large man will meet you.
 - b. You will meet some large man on the bridge.
- 4a. At least one large man will meet you.

- b. You met at least one large man on the bridge.

It is plausible to think that ‘a man’ occurs in (2) as a predicate in the *restricted quantifier* ‘ $\exists x$: x is a large man’. Since neo-Russellian readings as restricted quantifiers can be given for all complex denoting phrases discussed in “On Denoting,” such a modification might further rather than inhibit Russell’s philosophical agenda.

Illustrating the Distinction between Grammatical Form and Russellian Logical Form

5a. Everything is F

- b. Something is F
c. Nothing is F

6a. $\forall x Fx$

- b. $\sim \forall x \sim Fx$
c. $\forall x \sim Fx$

7a. Every G is F.

- b. All G’s are Fs.
c. Some G is F.
d. A G is F.
e. No G is F.

8a. $\forall x (x \text{ is } G \rightarrow x \text{ is } F)$

- b. $\forall x (x \text{ is } G \rightarrow x \text{ is } F)$
c. $\exists x (x \text{ is } G \ \& \ x \text{ is } F)$
d. $\exists x (x \text{ is } G \ \& \ x \text{ is } F)$
e. $\sim \exists x (x \text{ is } G \ \& \ x \text{ is } F)$

Russell’s Analysis of ‘The’

Russell’s analysis of (9) can be expressed in several equivalent ways, including (10), (11) and (12).

9. The father of Charles II was executed.

10. $\exists x [(Bxc \ \& \ \forall y (Byc \rightarrow y = x)) \ \& \ Ex]$

Someone who both begat Charles II and was identical with anyone who did so was executed – i.e. someone who was unique in begetting Charles II was executed.

11. $\exists x [Bxc \ \& \ Ex \ \& \ \forall y (Byc \rightarrow y = x)]$

Someone begat Charles II, was executed, and was identical with anyone who begat Charles II.

12. $\sim \forall x \sim [Bxc \ \& \ Ex \ \& \ \forall y (Byc \rightarrow y = x)]$

It is not always false of x that [x begat Charles II & x was executed & x was identical with anyone who begat Charles II.

Working with (12), we see that it expresses the proposition that a certain propositional function – call it p_{12} – is *not always false*. This propositional function – p_{12} – assigns to any object o the conjunction of: (i) the proposition that o begat Charles II, (ii) the proposition that o was executed, and (iii) the proposition expressed by ‘ $\forall y (Byc \rightarrow y = x)$ ’ relative to an assignment of o to the variable ‘ x ’. Proposition (iii), is the proposition that says of another propositional function, p_{iii} , that it is *always true* – where $p_{iii}(o^*)$ is the proposition that if o^* begat Charles II, then o^* is o itself. In short, proposition (iii) is the proposition that no one other than o begat Charles II. So, to say that p_{12} is not always false, and hence is sometimes true, is to say that for some object o , it is true that o begat Charles II, that no one else begat Charles II, and that o was executed. Since these are the truth conditions of (9), Russell’s analysis is truth-conditionally correct.

But does the analysis correctly identify the proposition actually expressed by (9)? Since Russell's theory of propositions allows truth-conditionally equivalent propositions to differ from one another, nothing we have said guarantees that it does. In fact, there is no evidence that Russell took this question seriously, or had anything credible to say about answering it. In addition to (10) – (12), the class of logically equivalent candidates for being the logical form of (9) includes (13) – (15).

13. $\exists x (Bxc) \ \& \ \exists x \forall y (Byc \rightarrow y = x) \ \& \ \forall x (Bxc \rightarrow Ex)$

Someone begat Charles II, at most one individual did so, and whoever did so was executed.

14. $\exists x [\forall y (x = y \rightarrow Byc) \ \& \ \forall y (Byc \rightarrow y = x) \ \& \ Ex]$

It is true of some individual that any individual he was identical with begat Charles II, that anyone who begat Charles II was identical with him, and that he was executed – in other words, since each thing is identical with itself (and only itself), some individual who was unique in begetting Charles II was executed.

15. $\exists x \forall y [(Byc \leftrightarrow y = x) \ \& \ Ex]$

It is true of some individual both that he was identical with any individual whatsoever iff that individual begat Charles II, and that he was executed. In other words, some individual who was unique in begetting Charles II was executed.

The analysis is intended to generalize to all cases in which a singular definite description occurs in a sentence [... the F ...]. The idea is to use a general rule R for translating an ordinary sentence containing a singular definite description into something more closely approximating its logical form.

R. $C [\text{the } F] \Rightarrow \exists x \forall y [(Fy \leftrightarrow y = x) \ \& \ Cx]$

R tells us that if a definite description occurs in a sentence along with additional material C, it can be eliminated (bringing us closer to the logical form of the sentence) by replacing the description with a variable, and introducing quantifiers plus the uniqueness clause as indicated. Russell comments that a sentence containing an occurrence of [the F] can be true only if F is true of one, and only one, thing. – This is vindicated by R, provided that R is understood in one particular, and restrictive, way.

The restriction concerns cases in which the description occurs in an embedded clause S' of a larger complex sentence S. The application of R needed to vindicate Russell's remarks is one in which C is understood as encompassing all of S, as opposed to applying solely within S'. Sentence (16), containing 'the present King of France' illustrates the point.

16. If France presently has one and only one King, then the King of France is among a dwindling number of European monarchs.

To apply R in the manner indicated is to let (17a) play the role of C in the rule, yielding (17b) as logical form.

17a. If France presently has one and only one king, then ___ is among a dwindling number of European monarchs

b. $\exists x \forall y [(y \text{ is presently a French King} \leftrightarrow y = x) \ \& \ \text{if France presently has one and only one King, then } x \text{ is among a dwindling number of European monarchs}]$

Since the truth (17b) requires France to presently have a King, (17b) is false, and so conforms to Russell's remarks.

However, this is not the only, or most obvious, way of understanding (16) – as Russell himself would emphasize. To get the Russellian reading in which (16) is true, we apply R, not to the conditional sentence as a whole, but only to its consequent clause, with 'is among a dwindling number of European monarchs' playing the role of C in that clause.

The result, (17c), is the second of the two logical forms of (16).

17c. If France presently has one and only one King, then $\exists x \forall y [(y \text{ is presently a French King} \leftrightarrow y = x) \& x \text{ is among a dwindling number of European monarchs}]$

Russell's Arguments Against other Theories

Having stated his analysis of [the F], Russell critiques alternative analyses, all of which lack what is for him the most crucial feature of the his analysis. That feature is his treatment of singular definite descriptions as “incomplete symbols”, which have “no meaning in isolation.” By this he means that although [the F] is a perfectly meaningful phrase, there is no entity, its meaning, that it contributes as a constituent to the propositions expressed by sentences containing it. This was also a feature of his analyses of other denoting phrases. But the gap between grammatical structure/constituency and propositional structure/constituency is far more dramatic for sentences containing [the F] than for sentences containing the others. Looking at the difference between (18a) and its Russellian logical form (18b), one can pinpoint the contributions of F and G to the proposition expressed. It is more complicated with ‘the’.

- 18a. The F is G
 b. $\exists x \forall y [(Fy \leftrightarrow y = x) \& Gx]$

Somehow, all this logical structure is the responsibility of ‘the’, even though no propositional constituent is its meaning. This is what Russell took to be his newly discovered truth.

It is also what he faulted alternative analyses for missing. He begins with criticisms of the theories of Meinong and Frege, both of which treat definite descriptions as contributing constituents – either objects denoted or meanings suited to denoting such objects – to the propositions expressed by sentences containing them. In each case, Russell finds details of the view that he (rightly) finds problematic. But these criticisms are only warm ups for the devastating critique he offers against *every possible theory (of every possible language) which treats singular definite descriptions as singular terms having meanings that (i) occur as constituents of propositions expressed by sentences containing them, and (ii) stand for objects they denote (which are referents of the descriptions themselves)*. Russell’s argument purports to uncover a conceptual incoherence in any theory of this sort. Since he seems to regard such a theory as the only possible serious competitor to the theory of “On Denoting,” he takes the demonstration of such incoherence to establish his new view. Thus, he prefaces his criticisms with the remark:

“The evidence for the above theory [of singular definite descriptions in “On Denoting”] is derived from the difficulties which seem unavoidable if we regard denoting phrases as standing for genuine constituents of the proposition in whose verbal expressions they occur.” (482)

Contra Meinong

Russell continues the above remark:

“Of the possible theories which admit such constituents the simplest is that of Meinong. This theory regards any grammatically correct denoting phrase as standing for an *object*. Thus “The present King of France,” “the round square,” etc., are supposed to be genuine objects. It is admitted that such objects do not *subsist*, but nevertheless they are supposed to be objects. This is in itself a difficult view; but the chief objection is that such objects, admittedly, are apt to infringe the law of contradiction. It is contended, for example, that *the existent present King of France exists*, and also does not exist; that the round square is round, and also not round; etc. But this is intolerable; and if any theory can be found to avoid this result, it is surely to be preferred.” (482-3)

Russell here calls attention to a difficulty in Meinong’s postulation of somehow real objects corresponding to all meaningful denoting phrases. Russell’s own earlier theory in *The Principles of Mathematics* suffered from a version of the problem, even if (charitably interpreted) it may have

avoided some of its worst excesses. Here he draws the line against any theory that assigns denoted objects to all meaningful descriptions.

Contra Frege

Turning to Frege, Russell says:

“The above breach of the law of contradiction is avoided by Frege’s theory. He distinguishes, in a denoting phrase, two elements, which we may call the *meaning* and the *denotation*. Thus, ‘the centre of mass of the Solar System at the beginning of the twentieth century’ is highly complex in *meaning*, but its *denotation* is a certain point, which is simple. ... One of the first difficulties that confront us, when we adopt the view that denoting phrases *express* a meaning and *denote* a denotation, concerns the cases in which the denotation appears to be absent. If we say “the King of England is bald,” that is, it would seem, not a statement about the complex *meaning* “the King of England,” but about the actual man denoted by the meaning. But now consider “the King of France is bald”. By parity of form, this also ought to be about the denotation of the phrase “the King of France”. But this phrase, though it has a *meaning* provided “the King of England” has a meaning, has certainly no denotation, at least in any obvious sense. Hence one would suppose that “the King of France is bald” ought to be nonsense; but it is not nonsense, since it is plainly false.” (483-4)

This, of course, misconstrues Frege, since on his view ‘The King of France is bald’ is not nonsense, but rather is perfectly meaningful, even though it is neither true nor false. Since sentences in this category express propositions correctly characterized as untrue, Russell has so far failed to identify any problem for Frege. Nor do his remarks about what statements of this sort are “about” help. There is nothing evidently wrong with the Fregean reply to Russell that although the statement that the King of England is bald is *about* a certain man (supposing that England has a King at the time of utterance), the statement that the King of France is bald is not *about* anyone. Russell’s seeming suggestion to the contrary is perhaps the remnant of the views expressed in his (1903) *Principles* that (i) denoting concepts, which are the meanings of denoting expressions, always denote, and (ii) propositions expressed by meaningful denoting phrases are always about the denotations of the denoting concepts they express – so that if nothing were denoted, there would be no meaning of the denoting phrase, and no proposition at all. Frege never held these views, and, if Russell himself ever strictly adhered to them, that time had passed.

Nevertheless, Russell does identify a genuine problem.

“Or again consider such a proposition as the following: “If u is a class which has only one member, then that one member is a member of u ,” or, as we may state it, “If u is a unit class, *the* u is a u ”. This proposition ought to be *always* true, since the conclusion is true whenever the hypothesis is true. But “the u ” is a denoting phrase, and it is the denotation, not the meaning, that is said to be a u . Now if u is *not* a unit class, “the u ” seems to denote nothing; hence our proposition would seem to become nonsense [actually truth valueless] as soon as u is not a unit class. Now it is plain that such propositions do *not* become nonsense [truth valueless] merely because their hypotheses are false. The King in “The Tempest” might say, “If Ferdinand is not drowned, Ferdinand is my only son”. Now “my only son” is a denoting phrase, which, on the face of it, has a denotation when, and only when, I have exactly one son. But the above statement would nevertheless have remained true, if Ferdinand had been in fact drowned.” (484)

Minor points aside, Russell is right. Frege’s theory wrongly characterizes certain *true* sentences containing non-denoting definite descriptions as *not true*. One such sentence is our earlier example (16), the Russellian analysis of which was given above.

16. If France presently has one and only one King, then the King of France is among a dwindling number of European monarchs.

Here, we assume with Frege and Russell that the sentences in question are material conditionals. For Frege, this means that their truth values are the result of applying the 2-place function f_{MC} that maps, the pair consisting of truth (of the antecedent) followed by falsity (of the consequent) onto the value falsity, while mapping all the other pairs of truth and/or falsity onto truth. Frege’s problem results

from his compositional theory of reference according to which (i) the truth value of an atomic sentence consisting of an n-place predicate P and n singular terms is the value – truth or falsity – assigned by the n-place function designated by P to the n arguments that are referents of the terms, (ii) definite descriptions are singular terms, (iii) truth and falsity are the only truth values, and (iv) all truth-functional connectives designate functions from n-tuples of truth values to truth values. Given (i) - (iii) Frege must characterize the consequent of (16) as neither true nor false, and so as having no truth value. Given (iv) he must characterize (16) in the same way: since f_{MC} assigns truth values only to *pairs* of truth values, one of which is missing in this case, there is no such thing as the truth value assigned to (16).

Though this problem is real, it is not obvious that the culprit is the Fregean analysis of descriptions. The problem could be avoided by modifying Frege’s system in several ways: e.g., (i) by assigning predicates sets of objects to which they apply, rather than functions from objects to truth values, and characterizing an atomic sentence as false whenever its n-tuple of terms fails to provide an n-tuple of referents that is a member of the set designated by its predicate, or (ii) by assigning non-denoting terms some entity to which the functions corresponding to predicates in the language always assign falsity (and over which the quantifiers do not range), or (iii) by expanding the number of truth values to include *neither-true-nor-false*, and adopting the truth function (19) for material conditionals, or (iv) by dispensing with functions as referents of truth-functional connectives, and characterizing the truth or falsity of truth functionally compound sentences using clauses like (20) in the theory of truth for the language.

19. *If A, then B*

Bl	T	F	*	‘*’ = <i>neither-true-nor-false</i>
A	T	F	*	
F	T	T	T	
*	*	*	*	

20. A conditional sentence [If A, then B] is false iff A is true and B is not; otherwise it is true.

The point is not that these fixes are equivalent (they aren’t), or even that there aren’t further objections against some of them (there are). The point is that without a great deal of further argument, Russell’s observation does not succeed in undermining Frege’s analysis of definite descriptions.

The “Gray’s Elegy” Argument: Contra the Possibility of Definite Descriptions as Singular Terms

The following words introduce Russell’s Master Argument against theories that treat definite descriptions as singular terms the meanings of which denote the objects they designate:

“The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong.

When we wish to speak about the *meaning* of a denoting phrase, as opposed to its denotation, the natural mode of doing so is by inverted commas. Thus we say:

The centre of mass of the Solar System is a point, not a denoting complex.
 “The centre of mass of the Solar System” is a denoting complex, not a point.

Or again,

The first line of Gray’s Elegy states a proposition.
 “The first line of Gray’s Elegy” does not state a proposition.

Thus taking any denoting phrase, say C, we wish to consider the relation between C and “C”, where the difference of the two is the kind exemplified in the above two instances.

We say, to begin with, that when *C* occurs it is the *denotation* we are speaking about; but when “*C*” occurs it is the *meaning*. Now the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation. But the difficulty which confronts us is that we cannot succeed in *both* preserving the connexion of meaning and denotation *and* preventing them from being one and the same; also that the meaning cannot be got at except by means of denoting phrases.” (485-6)

Here, Russell uses double quotes, not to produce a name of the expression that occurs inside them, but to produce (what is intended to be) a logically proper name of the meaning of that expression. The theories he criticizes are those that take [the *F*] to express a complex meaning (of which the meanings of ‘the’ and *F* are constituents) which, as a whole, denotes the unique thing of which *F* is true, if such there be. In saying “that when *C* occurs it is the *denotation* we are speaking about; but when “*C*” occurs it is the *meaning*,” he is saying that when a definite description *C* occurs in a sentence *S*, the proposition *S* expresses is one we use to talk about the object that its meaning denotes; but when a name consisting of double quotes, followed by *C*, followed by double quotes, occurs in a sentence *S*’, the proposition *S*’ expresses is one we use to talk about the meaning of *C*. That is our intention. But it is also what Russell believes he can show to be impossible.

It is impossible because it requires the meaning of *C* sometimes to occur in a proposition as a mere representative of its denotation – about which the proposition can be understood to say something – while occurring at other times, not as a representative of its denotation, but simply as that of which the proposition predicates something. The problem, Russell thinks, is that meanings can’t be ambiguous in this way. If definite descriptions have meanings that denote individuals, these meanings must either *always* represent their denotations, which the propositions in which they occur are about, or they must never do so, in which case the meanings can be subjects of predication in propositions containing them – but only by losing their ability to ever denote other things. This is what he means by saying “that we cannot succeed in *both* preserving the connexion of meaning and denotation *and* preventing them from being one and the same.” He adds in a final remark that the meaning *M* of a denoting phrase could, in principle, be made the subject of predication only in propositions that do not contain *M*, but rather contain a higher-order meaning *M** that denotes *M*. But he doesn’t, at this stage, say what is problematic about this.

The next four paragraphs of “On Denoting”, which spell out the argument, are among the most confusing that Russell ever wrote. Referred to by scholars as “the Gray’s Elegy passage,” they were, for 100 years, mostly dismissed as a distraction from the argument in “On Denoting.” This situation persisted until October of 2005, exactly 100 years to the month from the original appearance of the article in 1905. It was then, in a special issue commemorating the centenary of Russell’s original publication, that Nathan Salmon published “On Designating,” which untangled the confusions, and clarified the master argument Russell intended. I will simplify it here.

Let the expression ‘the first line of Gray’s Elegy’ be our example of a definite description expressing, according to the theories Russell wishes to refute, a complex meaning *M* that denotes sentence ‘the curfew tolls the knell of parting day’ (which is the first line of the Elegy). Next we let ‘*M*’ be a proper name of *M* – not a description, but a name that contributes its bearer to propositions expressed by sentences containing it. With this in mind, consider (21) and (22).

- 21a. The first line of Gray’s elegy is a sentence.
 - b. *M* is a sentence.
- 22a. The first line of Gray’s elegy denotes ‘the curfew tolls the knell of parting day’
 - b. *M* denotes ‘the curfew tolls the knell of parting day’

If the theories being investigated were correct, and our convention about what is to count as a name of meaning *M* were legitimate, then (21a) and (22b) would be true, while (21b) and (22a) should be

false. But this can be so only if (a) and (b) express different propositions. Russell's argument tries to show that these sentences can't do so.

What proposition is expressed by (21a)? It must be a proposition containing the property *being a sentence*, which is predicated of whatever is provided by its second constituent, M. Since M is, by hypothesis, a denoting concept, its role is to determine a denotation which, if it has the property predicated of it, will make the proposition true, and which, if it lacks the property, will make the proposition false. Knowing that the denotation of M is supposed to be a sentence, we judge proposition (21a) to be true. But now we are in for a surprise. What we didn't initially realize is that this line of reasoning would force us to take the proposition expressed by (21b) to be true, too. *Since the first and second constituents of proposition (21b) are the same as those of proposition (21a), and since the positions occupied by these constituents are the same, both the structure and the constituents of the two propositions are identical. It follows that the propositions don't differ at all, and so aren't two propositions, but only one.* The same conclusion holds for (22).

Despite this, the result seems absurd. Surely, if there are meanings that denote, we should be able to name them, and truly say of them – just as we do of other meanings – *that they are meanings*. But, Russell is convinced, acquiescence in the result (I) just reached does not allow this.

(I) (a) If p and p* are propositions that consist of the same n-place property the predication targets of which are provided by the same n-tuple of further constituents, which together exhaust the propositions, then p = p*. So, (b) If definite descriptions express meanings that denote unique objects satisfying them (if such objects there be), then these meanings can occur in propositions *only* in the role of presenting their denotations as the subjects of predication in the propositions; thus, these meanings can never themselves be the direct subjects of predication in any proposition in which they occur.

This, Russell thinks, prevents us from naming such meanings, and *saying of* them that they are meanings – or, indeed, from saying anything at all *of* them. The reason is (II), which he implicitly assumes.

(II) To *say of, believe of, or know of* an object o that it is so-and-so is to assert, believe, or know a singular proposition in which o occurs (in what Russell called in *The Principles of Mathematics* a “term accessible position.”)

Since (I) purportedly shows that there are no singular propositions of the sort mentioned in (II) in which a denoting concept occurs as direct subject of the proposition's predication, it follows that *one cannot say, believe, or know, of any such concept, that it is the meaning of anything*. But Russell also takes (III) to be an obvious truth about what is required in order for something to be the meaning of an expression.

(III) For any entity x whatsoever to be the meaning of an expression E for a group of speakers, it must be possible for those speakers to know *that E means x*, which is just to know the singular proposition in which the two-place relation *means* is directly predicated of the pair of E and x.

Given (I) – (III), Russell concluded that there are not, and could not be, meanings expressed by definite descriptions that denote the unique objects satisfying the descriptions. Since he took theories of this sort to be the only credible alternative to his theory that definite descriptions are “incomplete symbols” with “no meaning in isolation,” he concluded that his theory must be correct. His argument also generalized to all complex singular terms – e.g. to ‘2+3’ and ‘5²’ as well. For Russell, they too had to be treated as description, and hence as an *incomplete symbol*.

This argument has considerable force – particularly against Russell's earlier position in *The Principles of Mathematics*. Consider the first part of (III): for x to be the meaning of E, it must be possible to know *that E means x*. Surely that's right. What is it for one to know *that E means x*?

Russell would say that it is for one to bear the relation of knowing to the proposition that ‘E means x’ expresses, when the variable ‘x’ is treated as a logically proper name of the entity M that E means. Since variables are devices of pure reference, free of any descriptive information, he would maintain that ‘x’s contribution to the proposition p expressed by ‘E means x’ is simply M itself. Thus, he would conclude, p is the singular proposition in which the 2-place relation *means* is directly predicated of the pair of E and M, which is precisely what (III) tells us. All this seems right.

To understand (II), one must understand the contrast between believing or knowing *of x* that it is so-and-so, and believing or knowing *that D* is so-and-so, when ‘D’ is replaced by an arbitrary description that happens to denote x. To believe *of x* that it is so-and-so is to believe *that x is so and so* – which is to believe a singular proposition about x. This is different from believing *that D is so-and-so*, when ‘D’ is replaced by a description that denotes x. For many such descriptions, believing the resulting proposition is not *necessary* for believing the singular proposition – since, for many descriptions that x happens to satisfy, one can believe *that x is so-and-so* without having any idea whether the corresponding proposition *that x is D* is true. Russell would add that for many x-denoting replacements of ‘D’, believing *that D is so-and-so* is *not sufficient* for believing *that x is so-and-so*. For example, suppose that M is the meaning of E. It seems evident that even one who has no idea what E means, and doesn’t understand it, can know that it has a (single, unambiguous) meaning – and hence know *that the meaning of E is the meaning of E* – without knowing *that M is the meaning of E*. Again, this seems unassailable.

Locating the Source of the Problem

Still, Russell’s Gray’s Elegy argument is too good to be true. The leading culprit was Russell’s Platonic conception of propositions as structured complexes of objects and properties. *On this conception, the structure of a proposition is a hierarchical structure that organizes the pieces of the propositions into larger and larger substructures in a way analogous to the way in which grammatical structure organizes the constituents of a sentence into larger and larger subconstituents.* Just as a certain spot in grammatical structure is reserved for the predicate in the sentence, so a certain spot in propositional structure is reserved for the properties or relation being predicated of something. Consequently when two propositions have identical constituents arranged in identical structures, they must agree in truth value. In fact they must be the same proposition. This is what prevented Russell from seeing what is really going on in the Gray’s Elegy argument.

The key to getting at the truth about this is also the key to solving “the problem of the unity of the proposition” that puzzled Russell in *The Principles of Mathematics* and puzzled Frege in “On Concept and Object.” As I mentioned before when talking about that problem, the fundamental question is *How can propositions be bearers of intentionality – in ways in which mere lists, sets, n-tuples, functions, and formal tree-structures specifying hierarchical dominance and linear precedence relations, are not bearers of intentionality?* To answer that question, as well as the problem posed by the Gray’s Elegy argument, we have to give up the idea of propositions as platonic entities in which objects, properties, and functions somehow stand in purely abstract relations to one another. We have to replace it with a different conception that defines propositions in terms of the *cognitive operations of agents who entertain them* – cognitive operations of *focusing on, or thinking of, so-and-so, of applying a function to such-and-such, of predicating a property of an intended target, and the like.* When propositions are defined as complex acts in which agents perform these operations, it is natural (i) to take their constituents to be the objects, functions, and properties operated on, or with, and (ii) to view the structure a proposition as given by the cognitive operations required to entertain the proposition.

To see this, think of the difference between propositions (23a) and (23b), where *logicism* is a proper name for the proposition *that arithmetic is reducible to logic.*

- 23a. Russell attempted to establish that arithmetic is reducible to logic.
 b. Russell attempted to establish logicism.

These propositions predicate the same two place relation *attempting to establish*, of the same pair consisting of Russell and the proposition. But it is possible to believe the latter, without having any clue about Russell's views on arithmetic and hence without believing that *Russell attempted to establish that arithmetic is reducible to logic*. Susan might be in this situation after her first day of class in which her instructor has said that logicism is an important mathematical thesis that Russell tried to prove. If asked, what do you know about logicism, she could truthfully claim to know that Russell tried to establish logicism.

On the conception of propositions as cognitive acts or operations, the difference between propositions (23a) and (23b) is that whereas both require the agent to think of, and focus on, the man Russell and the proposition logicism, and to predicate *attempting to establish* of that ordered pair, only the former proposition imposes the further constraint that the agent think of, and focus on, its propositional argument *by entertaining it* – that is by predicating *being reducible to* of the pair *arithmetic* and *logic*.

In short, despite having identical truth conditions come from predicating the same property of the same arguments, the two propositions differ by virtue of the fact that only one requires the *constituents* of its propositional argument, logicism, to be cognitively accessed in the course of performing the operations in terms of which the larger proposition is defined. Whereas this constraint arises naturally on a conception of propositions as cognitive act types, the platonic conception of propositions that Russell worked with has no plausible way of accommodating it.

Now we go back to the Gray's Elegy cases. On an analysis that treats the description 'the first line of Gray's Elegy' as a singular term – both proposition (24a) and proposition (24b) are cognitive acts or event types in which an agent predicates *denoting 'the curfew tolls the knell of parting day'* of the something provided by a complex meaning M, which, we may imagine is f_{the} plus g (where g assigns to an arbitrary object o the proposition that o is a line of Gray's Elegy that precedes all other lines).

- 24a. The first line of Gray's elegy denotes 'the curfew tolls the knell of parting day'
 b. M denotes 'the curfew tolls the knell of parting day'

The difference between the two propositions is the sense in which that complex meaning provides the predication target. This difference has nothing to do with passive mental gazing at abstract platonic structure; instead it is a matter of *what the agent intends to be doing*. In the case of (24b) the agent intends to predicate *denoting 'the curfew tolls the knell of parting day'* of the complex meaning itself; in the case of (24a) the agent intends to predicate that content of *whatever is determined by that meaning*.

One can think of this as involving two closely related kinds of predication – one involving *direct predication* of the complex meaning (for which its constituents don't have to be cognitively accessed) and one involving *indirect predication* of the result of applying one of its constituents to the other (for which they do have to be cognitively accessed). *On this conception, the two propositions differ in structure and truth value, because they involve different, though related, predication operations on different, though related, predication targets – even though those targets are provided by the same constituent.*

Although this conception of propositions is not Russellian, it is one that can be reached by starting from a seminal idea in Russell's classic essay "On Truth and Falsehood" published in 1912 as chapter 12 of *The Problems of Philosophy*. Succumbing to his failure to solve the problem of propositional unity, he there abandons propositions entirely, proposing a *multiple-relation theory of judgment* to do the work that propositions had previously been intended to do. The seminal idea is that although abstract platonic propositions can never be unified, their constituents can be unified in judgment by

the mind of the agent doing the judging. What I am saying is that instead of giving up propositions he should have made the changes necessary to embrace them as cognitive acts of operations.